

Superposed Coherent States improve fidelity of NOON states generated in post-selection

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Abstract

Mixing the squeezed vacuum and coherent state in a beam-splitter generates NOON states of the electromagnetic field, but with fidelity less than unity. In this note it is shown that the output of the beam-splitter generates the NOON states with unit fidelity if superposed coherent states are used instead of the squeezed vacuum. Also, the post-selection probability of NOON states is shown to be higher.

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1 Introduction

Interferometers are useful in measuring the phase changes introduced by a non-opaque object. Usually, the object that changes the phase of the wave is introduced in one of the arms of the interferometer. The accuracy in the estimation of the relative phase $\delta\phi$ is limited by the standard quantum limit (SQL) $\delta\phi = 1/\sqrt{N}$, where N is the number of particles used in the measurement. The accuracy is improved by increasing the number of particles. However, quantum features make it possible to achieve higher accuracy than the SQL, for a given number of particles[1]. For instance, on using the nonclassical, squeezed light[2] in the input ports of Mach-Zehnder interferometer (MZI), the sensitivity approaches $1/N$ [3]. It has been realized that nonclassical correlations and entanglement are of relevance in surpassing the SQL[1, 4, 5]. In that context, an important class of bipartite entangled states that is useful is the NOON state[6, 7, 8, 9],

$$|\text{NOON}\rangle = \frac{1}{\sqrt{2}} [|N, 0\rangle + |0, N\rangle]. \quad (1)$$

Here $|N, 0\rangle$ and $|0, N\rangle$ correspond to the product states $|N\rangle|0\rangle$ and $|0\rangle|N\rangle$ respectively. It is possible to surpass the SQL in phase measurements if NOON states are used. Apart from their significance in phase measurements, the NOON states have been studied in the context of spatial super-resolution surpassing the Rayleigh limit[7], multiphoton interferometry[10] gravity wave detection[11] and quantum metrology in general[12, 13]. Because of their relevance in very many contexts, generation of optical NOON states has acquired a great significance. Many proposals have been put forth and some of them have been realized in the laboratories[14, 15, 16, 17]. Recently, these states have been generated by the method

of post-selection [18] on the output of a beam-splitter with properly chosen input states[19]. The output state is not exactly the NOON state, but the N-photon component of the output state is almost like a NOON state. This is a significant feature of this experiment. The quality of the output is expressed in terms of fidelity F , which is defined as the overlap between the NOON state and the normalized N-photon component of the output state from the beam-splitter. If $\sum_{n,m=0}^{\infty} C_{nm}|n, m\rangle$ is the bipartite state at the output of the beam-splitter, then

$$F = |\langle \text{NOON} | \sum_{n=0}^N C_{n,N-n} |n, N-n\rangle|^2 = \frac{|C_{0,N} + C_{N,0}|^2}{2}. \quad (2)$$

Parameters such as the amplitudes of the input states are optimized for each N to achieve maximum fidelity.

In the experiment of Afek *et al*[19], a beam-splitter is used to generate NOON state. The input for the beam splitter is the product state of the squeezed vacuum (SV) $|r\rangle$ and the coherent state (CS) $|\alpha\rangle$ of amplitudes r and α respectively. The number state expansions of these states are,

$$|r\rangle = \frac{1}{\cosh r} \sum_{n=0}^{\infty} (-\tanh r)^n \frac{\sqrt{(2n)!}}{2^n n!} |2n\rangle. \quad (3)$$

and

$$|\alpha\rangle = \exp(-\frac{|\alpha|^2}{2}) \sum_{n=0}^{\infty} \frac{\alpha^n}{\sqrt{n!}} |n\rangle. \quad (4)$$

Here, the amplitude r is real and α is complex. With $|r\rangle|\alpha\rangle$ as the input to the beam-splitter, the output contains all the NOON states as its components, in addition to non-NOON state components. The two input modes of the beam-splitter are denoted as a -mode and b -mode respectively.

2 Ideal input state to generate NOON states

In this section, the ideal input states for the experimental arrangement of [19] to generate NOON state by post-selection is discussed. The arrangement generates entanglement by a beam-splitter. The quantum state of the bipartite output from a lossless beam splitter is given by the action of the unitary operator,

$$U_{bs}(\gamma) = \exp(\gamma \hat{a} \hat{b}^\dagger - \gamma^* \hat{a}^\dagger \hat{b}), \quad (5)$$

on the input bipartite state[2]. Here $\gamma = i\pi/4$ for lossless, 50-50 beam splitter[2]. The operators \hat{a}^\dagger and \hat{a} are the creation- and annihilation-operators for the a -mode. The corresponding operators for the b -mode are \hat{b}^\dagger and \hat{b} respectively. The suitable input states for the beam-splitter to generate NOON states at the output are obtained by observing that

$$|\text{NOON}\rangle = U_{bs}(\gamma) U_{bs}^\dagger(\gamma) |\text{NOON}\rangle, \quad (6)$$

where the adjoint operator $U_{bs}^\dagger(\gamma) = U_{bs}(-\gamma)$. This relation implies that the apt input to produce NOON states is $U_{bs}(-\gamma) |\text{NOON}\rangle$. To analyze further, the bipartite number state expansion of the input state is useful. To this end, it may be noted that the operators $\hat{a}^\dagger \hat{b}$, $\hat{a} \hat{b}^\dagger$ and $\hat{a}^\dagger \hat{a} - \hat{b}^\dagger \hat{b}$ form a closed algebra under commutation. Defining $K_+ = \hat{a}^\dagger \hat{b}$, $K_- = \hat{a} \hat{b}^\dagger$ and $K_0 = (\hat{a}^\dagger \hat{a} - \hat{b}^\dagger \hat{b})/2$, we have

$$[K_\pm, K_0] = \pm K_\pm, \quad [K_-, K_+] = K_0. \quad (7)$$

These commutation relations define the SU(2) algebra[20, 21]. Consequently, the operator $U_{bs}^\dagger(\gamma)$ has the following disentangled forms[22]

$$U_{bs}^\dagger(\gamma) = \exp(p\hat{a}^\dagger\hat{b}) \exp[q(\hat{a}^\dagger\hat{a} - \hat{b}^\dagger\hat{b})] \exp(r\hat{a}\hat{b}^\dagger) \quad (8)$$

$$= \exp(p\hat{a}\hat{b}^\dagger) \exp[q(\hat{b}^\dagger\hat{b} - \hat{a}^\dagger\hat{a})] \exp(r\hat{a}^\dagger\hat{b}), \quad (9)$$

Setting $\gamma = |\gamma| \exp(-i\theta)$, the parameters p , q and r are $\exp(-i\theta) \tan |\gamma|$, $2 \log \sec |\gamma|$ and $-\exp(-i\theta) \tan |\gamma|$ respectively. Employing the disentangled form of the unitary operator $U_{bs}^\dagger(\gamma)$, the number state expansion of the required input state is obtained to be

$$U_{bs}^\dagger|\text{NOON}\rangle = \frac{\exp(-qN/2)}{\sqrt{2}} \sum_{k=0}^N \sqrt{\binom{N}{k}} (p^k + p^{N-k}) |N-k, k\rangle. \quad (10)$$

This is the input state that a beam-splitter converts to a NOON state. It is of interest to note that the required input state can be viewed as a superposition of two SU(2) coherent states: one generated from the state $|N, 0\rangle$ and the other generated from the state $|0, N\rangle$.

For a 50-50, lossless, symmetric beam-splitter, $\gamma = i\pi/4$ and the values of p , q and r are $-i$, $\log 2$ and $-i$ respectively. With this choice of γ ,

$$U_{bs}^\dagger(i\frac{\pi}{4})|\text{NOON}\rangle = \frac{1}{\sqrt{2}^{N-1}} \sum_{k=0}^{N/2} (-1)^k \sqrt{\binom{N}{2k}} |N-2k, 2k\rangle, \quad (11)$$

if N is an even multiple of two, for example, $N = 4, 8, 12, \dots$. If N is an odd multiple of two $[N = 2(2m+1)]$, then the required input state is

$$U_{bs}^\dagger(i\frac{\pi}{4})|\text{NOON}\rangle = \frac{1}{\sqrt{2}^{N-1}} \sum_{k=0}^{N/2-1} (-1)^k \sqrt{\binom{N}{2k+1}} |N-2k-1, 2k+1\rangle. \quad (12)$$

It is to be pointed out that if N is an odd multiple of two, then the required input involves bipartite number states with respective odd numbers of photons. Such a combination is not possible if the SV is used as one of the input fields since the state is a linear superposition of even number states only. The finite superposition given in Eqns. 11 and 12 are the respective input states to generate the NOON states corresponding to even and odd multiples of two.

But these finite superpositions are difficult to generate experimentally for large N . Alternately, it is possible to use bipartite states of the electromagnetic field whose N -photon component is similar to the form of the ideal input states. This is the idea suggested in [18] and subsequently realized[19]. For proper choice of the amplitudes of the input states, the maximum achievable fidelity is 0.94. This has been experimentally implemented to generate NOON states upto $N = 5$.

3 Superposed coherent states versus squeezed vacuum

It is interesting to see if the fidelity can be increased to unity by a suitable choice of input states. This, in turn, means that the bipartite input to the beam-splitter should have N -photon component in the form given in Eq. 11 and Eq. 12 depending on whether N is an even or odd multiple of two[23]. Like the SV, even coherent states (ECS) are constructed as a superposition of the even number states [24, 25, 26] of the electromagnetic field. The ECS, denoted by $|\beta, +\rangle$, of complex amplitude β are obtained as the symmetric combination of the coherent states $|\beta\rangle$ and $|- \beta\rangle$,

$$|\beta, +\rangle = \frac{1}{\sqrt{\cosh |\beta|^2}} \sum_{n=0}^{\infty} \frac{\beta^{2n}}{\sqrt{(2n)!}} |2n\rangle. \quad (13)$$

Similarly, odd coherent states (OCS), denoted by $|\beta, -\rangle$, are defined as the antisymmetric combination of the coherent states of amplitudes β and $-\beta$,

$$|\beta, -\rangle = \frac{1}{\sqrt{\sinh |\beta|^2}} \sum_{n=0}^{\infty} \frac{\beta^{2n+1}}{\sqrt{(2n+1)!}} |2n+1\rangle. \quad (14)$$

Both the ECS and OCS have been studied in the context of the optical Schrodinger cat states. In addition, these states possess many other nonclassical features such as squeezing, sub-Poissonian statistics, etc. Of importance in the present context is that such superpositions of the propagating light fields can be generated in experiments [27, 28, 29, 30].

Now, the suitability of the ECS and OCS in the NOON state generation is discussed. The N -photon component of the product state $|\beta, +\rangle|\alpha\rangle$ is

$$|\beta+, \alpha\rangle_{N,e} \propto \sum_{n=0}^{N/2} \sqrt{\binom{N}{2k}} \left[\frac{\alpha}{\beta}\right]^{2k} |N-2k, 2k\rangle, \quad (15)$$

if N is even, while it is

$$|\beta+, \alpha\rangle_{N,o} \propto \sum_{k=0}^{N/2-1} \sqrt{\binom{N}{2k+1}} \left[\frac{\alpha}{\beta}\right]^{2k+1} |N-2k-1, 2k+1\rangle, \quad (16)$$

if N is odd. If the input is the OCS-CS $|\beta, -\rangle|\alpha\rangle$, the N -photon components corresponding to even N is

$$|\beta-, \alpha\rangle_{N,e} \propto \sum_{k=0}^{N/2} \sqrt{\binom{N}{2k+1}} \left[\frac{\alpha}{\beta}\right]^{2k+1} |N-2k-1, 2k+1\rangle, \quad (17)$$

while the odd N component is

$$|\beta-, \alpha\rangle_{N,o} \propto \sum_{n=0}^{(N-1)/2} \sqrt{\binom{N}{2k}} \left[\frac{\alpha}{\beta}\right]^{2k} |N-2k, 2k\rangle. \quad (18)$$

Comparison of Eqs. 15 and 18 with Eqs. 11 and 12 respectively makes it clear that the N -photon component of the input states are precisely of the required form to generate the NOON states if $\alpha = i\beta$. This, in turn, means that the NOON state fidelity of the output state is unity. Thus, ideal NOON states of even N can be obtained by mixing coherent states with Schrodinger cat states. The discussion so far has been restricted to generating those NOON states where N is even. It may be noted that with the choice of γ used here, the fidelity of NOON state is always less than unity if N is odd.

Though the fidelity defined earlier gives a measure of the "NOON"ness of the N -photon component of the output state, the probability of NOON state occurrence is given by the overlap between the input state to the beam-splitter and the ideal input state. This quantity can also be viewed as the overlap between the NOON state and the realized output state. As indicated before, the ideal input state to NOON states with N as an even multiple of two is contained in the state $|\alpha+\rangle|i\alpha\rangle$. The corresponding overlap is

$$|\langle i\alpha|\langle\alpha, +|U_{bs}^\dagger(i\frac{\pi}{4})|\text{NOON}\rangle|^2 = \frac{2^{N-1}|\alpha|^2 \exp(-|\alpha|^2)}{N! \cosh |\alpha|^2}. \quad (19)$$

If N is an odd multiple of two, the required input state is contained as a component of the input state $|\alpha, -\rangle|i\alpha\rangle$. In this case, the overlap of the ideal input with the actual input is

$$|\langle i\alpha|\langle\alpha, -|U_{bs}^\dagger(i\frac{\pi}{4})|\text{NOON}\rangle|^2 = \frac{2^{N-1}|\alpha|^2 \exp(-|\alpha|^2)}{N! \sinh |\alpha|^2}. \quad (20)$$

In Fig. 1 the dependence of the overlap on the amplitude α is shown. The figures corresponding to $N = 2$ and $N = 6$ depict the overlap for the OCS-CS with the ideal input and that of the SV-CS with the ideal input. It is seen that the overlap for the former is orders of magnitude larger than the later. If $N=4$ or 8 , the overlap of the ideal input state with the ECS-CS combination is still larger than that in the SV-CS case. In the context of experiments, these results imply that the probability of NOON state generation is correspondingly higher. It may be pointed out that the overlap in SV-CS case is orders of magnitude smaller than the other two cases, that the corresponding curve does not appear to deviate from zero due to the scale used in the figures. Also, the peak values of the overlap occur for progressively larger values of $|\alpha|$ as N increases. Therefore, by properly choosing the amplitude α , it is possible to maximize the post-selection probability for a particular NOON state.

4 Summary

The NOON state fidelity of the beam-splitter output is enhanced by using the even/odd coherent states and the coherent states in the input. The even coherent states are suitable to produce the NOON states if N is an even multiple of two, while the odd coherent states are the apt state if N is an odd multiple of two. The probability of NOON state detection can be optimized by proper choices for the amplitudes of the input coherent state and the even/odd coherent states. Compared to the case of using the squeezed vacuum in the

input, the probability of NOON state detection by post-selection is higher if superposed coherent states are used. The possibility of higher fidelity makes allowance for the non-ideal characteristics of the various optical components used in the experiments. In conclusion, unit-fidelity and high-probability generation of the NOON states is better achieved by using the even/odd coherent states.

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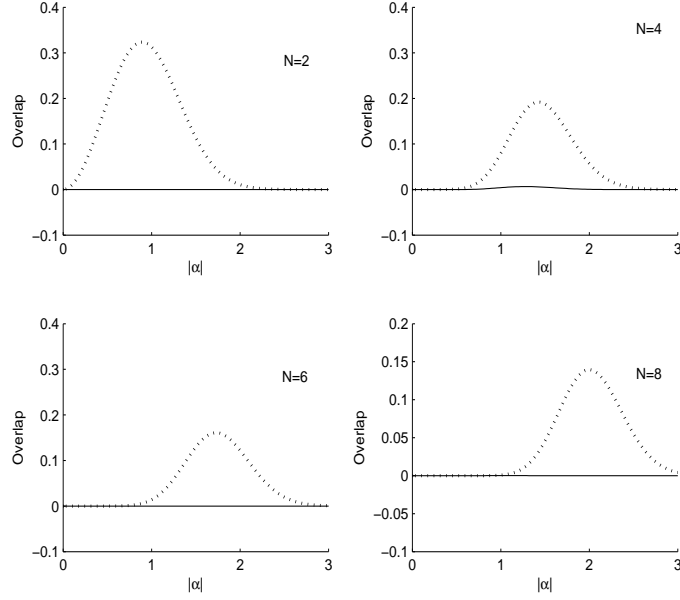


Figure 1: Overlap of NOON state and the output from the beam-splitter as a function of the magnitude of the amplitude α , for different values of N . For $N = 2$ and $N = 6$, the SV-CS (continuous) and the OCS-CS (dash) cases are shown. For $N = 4$ and $N = 8$, the ECS-CS (dash) and the SV-CS (continuous) are shown. The overlap in the SV-CC case is nonzero which, because of comparatively smaller magnitude than the other cases, appears to be nearly zero. Though the overlap is nonnegative, for the purpose of depicting near-zero values, the scale of the ordinate is extended to include negative values.